



Property Risk Consulting Guidelines

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LIQUID HOLDUP ESTIMATION

INTRODUCTION

When an emergency occurs, chemical plants typically initiate a shutdown and close the emergency block valves to isolate the affected unit or piece of equipment. If the emergency in question is an equipment breach and spill fire, this action limits the fire's fuel supply to the amount of liquid already spilled plus the liquid holdup of the isolated equipment that has suffered the breach. Since designing drainage systems, deluge systems, diking, fireproofing, equipment spacing, etc. all require knowledge of the expected spill size, the ability to estimate the liquid holdup in process equipment and storage vessels is vital.

If materials in the process pose a vapor cloud explosion potential, it is equally important to know the quantities involved so that the possible blast overpressures can be calculated. These in turn enable management to select proper unit location and design adequate blast resistance into the process structures.

POSITION

Before spending large amounts of time to measure all the equipment and perform the calculations, check to see if the information is already available. Manufacturers, designers, operators or maintenance personnel often have this information. It can also be found on some plant process and instrument diagrams (P&IDs). If the information is not available, then calculate the liquid holdup by doing the following.

- Calculate from block valve to block valve. This requires an assumption as to which block valves can realistically be closed in a fire emergency. A remotely operated valve within the fire area or a manually operated one outside the area could probably be closed after a fire starts. A manually operated valve located well inside the fire area will probably be inaccessible.
- Obtain the dimensions of the equipment involved.
- Check the service or normal operating conditions. Some equipment is usually full, some partly filled, and some nearly empty.
- Calculate the theoretical maximum liquid holdup of the equipment, which is the internal geometric volume.
- Deduct the volume of any permanent internal obstructions, such as agitators, baffles, trays, catalyst beds, etc.
- Deduct the volume of the minimum expected (designed) void spaces. For example, a distillation tower or a decanter would not normally be filled with liquid but a storage tank could be.

- Make allowances for the heels in columns or drip legs on receivers.

DISCUSSION

Certain approximations are useful for the purposes of calculating the internal volume of equipment. Most equipment shapes can be more easily calculated if they are assumed to be combinations of simple shapes such as rectangular blocks, spheres or cylinders. In all the formulas for common shapes shown below, V = volume, L = length, W = width, H = height of liquid and D = diameter.

Rectangular Box

$$V = L \times W \times H$$

Cylinder (Vertical)

$$V = \frac{\pi}{4} (H \times D^2)$$

Cylinder (horizontal with flat ends)

$$V = L \times \frac{D^2}{8} \left[2 \arccos \left(1 - \frac{2 \times H}{D} \right) - \frac{8}{D^2} \left(\frac{D}{2} - H \right) \sqrt{(D \times H) - H^2} \right]$$

Note that this is for a partially filled vessel. If the vessel is full, the equation simplifies to:

$$V = \frac{\pi}{4} \times L \times D^2$$

Cylinder (horizontal with hemispherical ends)

$$V = \left\{ (L - D) \times \frac{D^2}{8} \left[2 \arccos \left(1 - \frac{2 \times H}{D} \right) - \frac{8}{D^2} \left(\frac{D}{2} - H \right) \sqrt{(D \times H) - H^2} \right] \right\} + \left\{ \frac{\pi}{3} \times H^2 \left(\frac{3 \times D}{2} - H \right) \right\}$$

Note that this is for a partially filled vessel. If the vessel is full, the equation simplifies to:

$$V = \frac{\pi}{12} \times D^2 \times (3L - D)$$

Tilted Cylinder

Determining the volume of a tilted cylinder is more complicated than a horizontal or vertical cylinder.

Consider cylindrical tank with radius r and length L is tilted at angle θ . The tank contains liquid at height h measured at the high end of the tank. What is the liquid volume?

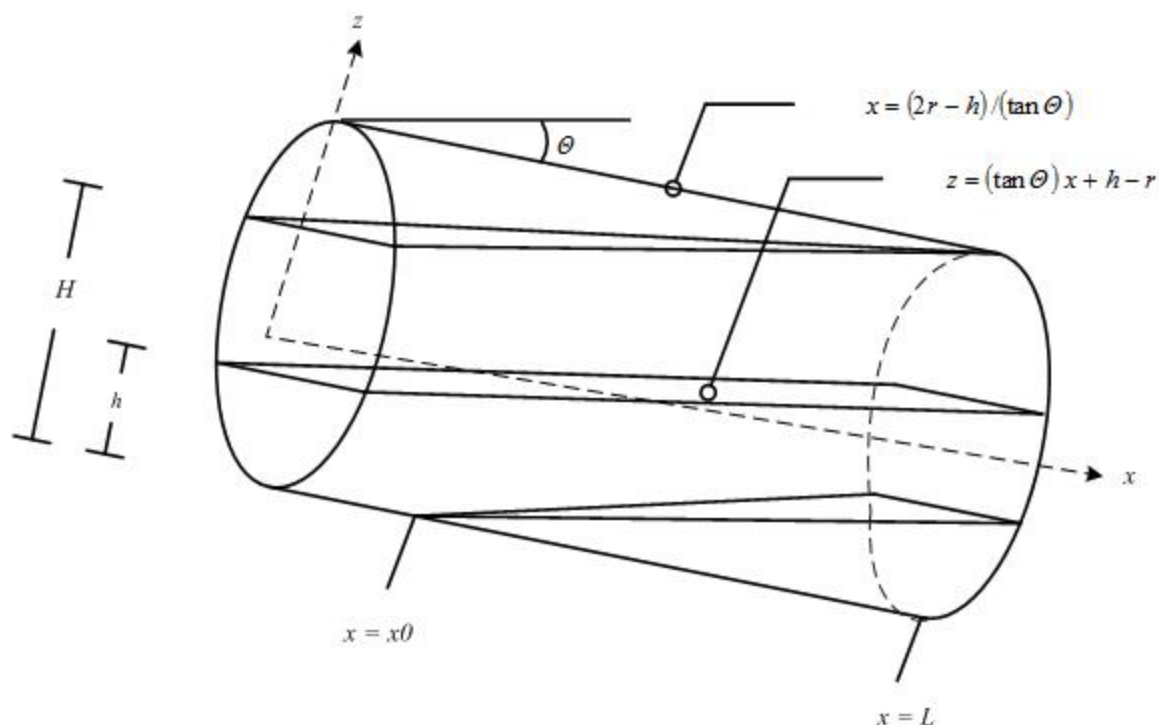


Figure 1.

First divide the volume into thin slices of area parallel to the ends of the tank. A representative slice of area looks like this:

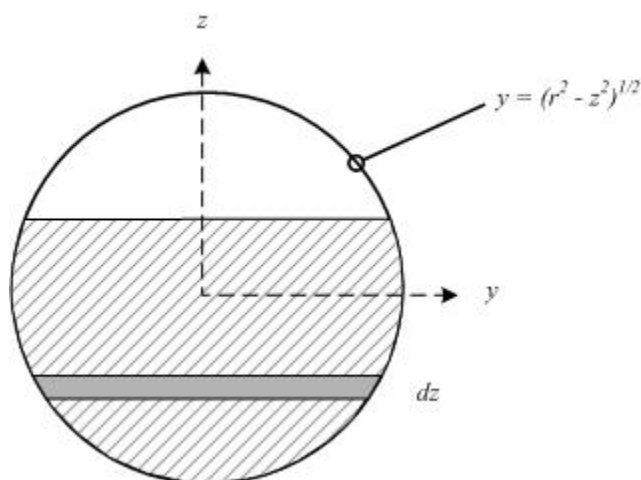


Figure 2.

The area of a slice is:

$$A = \int_{z=-r}^{z=(\tan \Theta)x+h-r} 2y \, dz = 2 \int_{z=-r}^{z=(\tan \Theta)x+h-r} \sqrt{r^2 - z^2} \, dz$$

$$= 2 \left(\frac{1}{2} \right) \left[z \sqrt{r^2 - z^2} + r^2 \sin^{-1} \left(\frac{z}{r} \right) \right]_{z=-r}^{z=(\tan \Theta)x+h-r}$$

$$= [(\tan \Theta)x + h - r] \sqrt{r^2 - [(\tan \Theta)x + h - r]^2} + r^2 \sin^{-1} \left[\frac{(\tan \Theta)x + h - r}{r} \right] - r^2 \sin^{-1}(-1)$$

The volume breaks into three cases: $0 \leq h \leq H$, $-L(\tan \Theta) < 0$ and $H < h \leq 2r$. The first case applies whenever the top of the liquid surface touches both ends of the tank. This would probably be the most common case. The second case applies from just over empty to when the liquid surface reaches the bottom of the higher end. The third case covers from when the liquid surface reaches to top of the lower end to when the tank is full.

Case 1: $0 \leq h \leq H$

$$V = \int_{x=0}^{x=L} \left[[(\tan \Theta)x + h - r] \sqrt{r^2 - [(\tan \Theta)x + h - r]^2} + r^2 \sin^{-1} \left[\frac{(\tan \Theta)x + h - r}{r} \right] + \frac{\pi r^2}{2} \right] dx$$

Now let: $u = (\tan \Theta)x + h - r$

$$du = \tan \Theta dx$$

$$\begin{aligned} V &= \int_{x=0}^{x=L} \left[u \sqrt{r^2 - u^2} + r^2 \sin^{-1} \left(\frac{u}{r} \right) + \frac{\pi r^2}{2} \right] \cot \Theta du \\ &= \cot \Theta \left[\left(-\frac{1}{3} \right) \sqrt{(r^2 - u^2)^3} + r^2 \left(u \sin^{-1} \left(\frac{u}{r} \right) + \sqrt{r^2 - u^2} \right) + \left(\frac{\pi r^2}{2} \right) u \right]_{x=0}^{x=L} \end{aligned}$$

Substituting for u:

$$\begin{aligned} V &= \cot \Theta \left[\left(-\frac{1}{3} \right) \sqrt{(r^2 - [(\tan \Theta)L + h - r]^2)^3} + r^2 [(\tan \Theta)L + h - r] \sin^{-1} \left(\frac{(\tan \Theta)L + h - r}{r} \right) \right. \\ &\quad \left. + r^2 \sqrt{r^2 - [(\tan \Theta)L + h - r]^2} + \frac{\pi r^2}{2} [(\tan \Theta)L + h - r] \right]_{x=0}^{x=L} \end{aligned}$$

Substituting the limits for x:

$$\begin{aligned} V &= \cot \Theta \left[\left(-\frac{1}{3} \right) \sqrt{(r^2 - [(\tan \Theta)L + h - r]^2)^3} + r^2 [(\tan \Theta)L + h - r] \sin^{-1} \left(\frac{(\tan \Theta)L + h - r}{r} \right) \right. \\ &\quad \left. + r^2 \sqrt{r^2 - [(\tan \Theta)L + h - r]^2} + \frac{\pi r^2}{2} [(\tan \Theta)L + h - r] + \left(\frac{1}{3} \right) \sqrt{(r^2 - (h - r)^2)^3} \right. \\ &\quad \left. - r^2 (h - r) \sin^{-1} \left(\frac{h - r}{r} \right) - r^2 \sqrt{r^2 - (h - r)^2} - \frac{\pi r^2}{2} (h - r) \right] \end{aligned}$$

NOTE: In this solution, the height of the liquid, h , is measured at and parallel to the higher tank end (see drawing). This measurement could be easily read from an external manometer-type gauging tube and float. Measuring the liquid height at and parallel to the lower end of the tank would also determine x_0 as used in Case 2. However, any other way of measuring the liquid height results in a value that can be converted to the value of h used in this solution.

Case 2: $-L(\tan \Theta) < h < 0$

This is the same as Case 1, except that the lower limit for x is increased to x_0 (see drawing).

Using the new limit:

$$\begin{aligned}
 V = & \cot \Theta \left[\left(-\frac{1}{3} \right) \sqrt{r^2 - [(\tan \Theta)L + h - r]^2}^3 + r^2 [(\tan \Theta)L + h - r] \sin^{-1} \left(\frac{(\tan \Theta)L + h - r}{r} \right) \right. \\
 & + r^2 \sqrt{r^2 - [(\tan \Theta)L + h - r]^2} + \frac{\pi r^2}{2} [(\tan \Theta)L + h - r] + \left(\frac{1}{3} \right) \sqrt{r^2 - [(\tan \Theta)x_0 + h - r]^2}^3 \\
 & - r^2 [(\tan \Theta)x_0 + h - r] \sin^{-1} \left(\frac{(\tan \Theta)x_0 + h - r}{r} \right) - r^2 \sqrt{r^2 - [(\tan \Theta)x_0 + h - r]^2} \\
 & \left. - \frac{\pi r^2}{2} [(\tan \Theta)x_0 + h - r] \right]
 \end{aligned}$$

Case 3: $H < h \leq 2r$

This is the same as Case 1, except that the upper limit for x is decreased to where the liquid first touches the top of the tank. This happens when h exceeds $H = 2r - L(\tan \Theta)$. The upper limit then used is $x = (2r - h)/(\tan \Theta) = M$. An additional term is added for the length of the trunk where liquid touches the top. Using the new limit and term:

$$\begin{aligned}
 V = & \cot \Theta \left[\left(-\frac{1}{3} \right) \sqrt{r^2 - [(\tan \Theta)M + h - r]^2}^3 + r^2 [(\tan \Theta)M + h - r] \sin^{-1} \left(\frac{[(\tan \Theta)M + h - r]}{r} \right) \right. \\
 & + r^2 \sqrt{r^2 - [(\tan \Theta)M + h - r]^2} + \frac{\pi r^2}{2} [(\tan \Theta)M + h - r] + \left(\frac{1}{3} \right) \sqrt{r^2 - (h - r)^2}^3 \\
 & \left. - r^2 (h - r) \sin^{-1} \left(\frac{h - r}{r} \right) - r^2 \sqrt{r^2 - (h - r)^2} - \frac{\pi r^2}{2} (h - r) \right] + \pi r^2 (L - M)
 \end{aligned}$$

Sphere

$$V = \frac{\pi}{3} \times H^2 \left(\frac{3 \times D}{2} - H \right)$$

Note that this is for a partially filled vessel. If the vessel is full, the equation simplifies to:

$$V = \frac{\pi}{6} \times D^3$$

Pipes

The volume contained within pipes should not be ignored. The holdup volume of a unit composed of a large vessel and short pipe runs is dominated by the vessel volume, but many units are more complicated. Long pipe runs, many parallel runs or large pipe diameters can quickly push the pipe holdup to appreciable levels. This is particularly true if the unit is deficient in emergency block valves.

The internal volume of piping can be calculated using the formula for a filled cylinder with flat ends. Keep in mind, however, the nominal internal diameter and the true internal diameter of the pipe are different. This can be significant for the smaller pipe sizes. Table 1 contains the volume to length values for the most common pipe sizes.

Odd Shapes

Equipment that is oddly shaped so as to make volume calculations impractical can be addressed directly by filling it with water or other inert liquid and directly measuring the volume used. Keep in

mind that water is considerably heavier than most hydrocarbons so the vessel and its supports must be designed to bear the additional weight for this approach to be practical.

TABLE 1
Pipe Volumes

Nominal Diameter in. (mm)	Capacity Sch 40 gal/ft (L/m)	Sch 80 gal/ft (L/m)
1 (25)	0.045 (0.56)	0.037 (0.46)
1.25 (32)	0.078 (0.97)	0.067 (0.83)
1.5 (40)	0.106 (1.32)	0.092 (1.14)
2 (50)	0.174 (2.16)	0.153 (1.90)
2.5 (65)	0.248 (3.08)	0.220 (2.73)
3 (80)	0.383 (4.76)	0.343 (4.26)
3.5 (90)	0.513 (6.38)	0.462 (5.74)
4 (100)	0.660 (8.20)	0.597 (7.41)
5 (125)	1.040 (12.92)	0.945 (11.74)
6 (150)	1.501 (18.64)	1.354 (16.82)
8 (200)	2.660 (33.04)	2.372 (29.46)
10 (250)	4.096 (50.87)	3.730 (46.32)
12 (300)	5.815 (72.22)	5.278 (65.55)
14 (350)	7.027 (87.27)	6.375 (79.17)
16 (400)	9.180 (114.01)	8.357 (103.79)
18 (450)	11.620 (144.31)	10.607 (131.73)
20 (500)	14.439 (179.32)	13.128 (163.04)
24 (600)	20.883 (259.35)	18.969 (235.58)